Quantum Mechanics as Topological Intersection Theory: The Born Rule, Wavefunction Collapse, and Planck's Constant from Worldline Non-Injectivity

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April 2026

Abstract

We derive the foundations of quantum mechanics from the geometry of non-injective worldlines within the TPST–DGQ framework. The Born rule $|\psi(x,t)|^2$ is shown to be not a postulate but a theorem: it is the normalised density of intersections of a single ultra-relativistic worldline $X^\mu(\tau)$ with the constant-time hypersurface Σ_t. The wavefunction $\psi(x,t)$ is the square root of this intersection density, and its phase is the proper-time transport of the physical state between consecutive intersection points. Planck's constant $\hbar$ is derived from the geometry of consecutive worldline folds: it is the minimum action per radian of phase for a fold to be stable against merging, expressed as $\hbar = mc\epsilon/(2\pi\gamma_{\rm crit})$, where ϵ is the UV cutoff and $\gamma_{\rm crit}$ is the critical Lorentz factor. The quantum potential $Q = -(\hbar^2/2m)\nabla^2\sqrt{\rho}/\sqrt{\rho}$ is derived from the inter-sheet electromagnetic interaction between the N topological sheets, as established by the Maxwell Topological Emergence Identity of the companion paper. The Schrödinger equation then follows as the transport law for the intersection density under conditions of inter-sheet phase coherence, which is itself a theorem of the Topological Emergence Identity. Wavefunction collapse is the topological transition $N > 1 \to N = 1$ produced by a measurement interaction that reduces the Lorentz factor below $\gamma_{\rm crit}$. The double-slit interference pattern is a direct consequence of inter-sheet electromagnetic interference, requiring no additional postulates. The same identity $N(\epsilon)\cdot\epsilon^{+(d-2)} = O(1)$ that regularises the Ryu–Takayanagi entropy and the Coulomb self-energy is shown to regularise $|\psi|^2$ as well, unifying quantum mechanics, classical electrodynamics, and holographic gravity under a single topological principle.

1 Introduction

The foundations of quantum mechanics rest on postulates that have never been derived from deeper principles.

The Born rule states that $|\psi(x,t)|^2$ is the probability of finding a particle at position x at time t. Its physical origin is unknown.

The superposition principle states that a particle can exist in multiple states simultaneously. No physical mechanism explains why a particle occupies multiple positions before measurement.

Wavefunction collapse is an instantaneous, irreversible, non-unitary reduction to a single eigenstate upon measurement. The measurement problem has remained open for a century.

The present paper derives all three from a single geometric fact: a worldline $X^\mu(\tau)$ with Lorentz factor $\gamma > \gamma_{\mathrm{crit}}$ intersects the constant-time hypersurface Σ_t in $N(\epsilon) > 1$ distinct spatial points.

The companion papers establish the following results, which are used here without reproof:

(i) The intersection multiplicity scales as $N(\epsilon) \sim \epsilon^{-(d-2)}$ (Lemma 2 of [1]).

(ii) The topological average $N(\epsilon)^{-1} \sum_i f_i = O(1)$ whenever $f_i \sim \epsilon^{-(d-2)}$ (Lemma 3 of [1]).

(iii) The same cancellation operates in classical electrodynamics via the Maxwell Topological Emergence Identity, regularising the Coulomb self-energy without external cutoff (Section 12 of [1]).

The logical structure of the paper is as follows. Section 2 derives the Born rule. Section 3 defines the wavefunction and its phase. Section 4 derives Planck's constant from the geometry of worldline folds. Section 5 derives the quantum potential from inter-sheet electromagnetic interaction and shows that together with the continuity equation it produces the complete Schrödinger equation. Section 6 proves that inter-sheet phase coherence is not a postulate but a theorem of the Topological Emergence Identity. Section 7 derives wavefunction collapse as a topological transition. Section 8 explains the double-slit experiment without additional postulates. Section 9 presents the universal cancellation identity.

2 The Born Rule as a Theorem

2.1 The topological intersection density

Let $X^\mu(\tau) = (X^0(\tau), \mathbf{X}(\tau))$ be a timelike worldline with Lorentz factor $\gamma > \gamma_{\text{crit}}$, satisfying

$$\exists\, t \in \mathbb{R} : \quad \#\{\tau \mid X^0(\tau) = t\} = N(\epsilon) > 1. \tag{1}$$

The N intersection points with Σ_t are $\{\mathbf{X}_1(t), \ldots, \mathbf{X}_N(t)\}$. Define the *topological intersection density*:

$$\rho(x,t) := \frac{1}{N(\epsilon)} \sum_{i=1}^{N(\epsilon)} \delta^{(3)}\big(\mathbf{x} - \mathbf{X}_i(t)\big). \tag{2}$$

2.2 Normalisation

$$\int_{\mathbb{R}^3} \rho(x,t)\, d^3x = \frac{1}{N(\epsilon)} \sum_{i=1}^{N(\epsilon)} 1 = 1. \tag{3}$$

The density is automatically normalised by the Ontological Identity Principle: the N intersections are N appearances of the same physical entity, so the total weight is 1. This follows from the same principle that ensures $\partial_\mu J^\mu_{\text{DGQ}} = 0$ in the Maxwell case and $\delta S_{\text{DG}} = \delta\langle \hat{K}_B \rangle$ in the holographic case.

2.3 Born rule

The Born rule $|\psi(x,t)|^2 = \rho(x,t)$ is a geometric frequency: the fraction of the N topological sheets that occupy position x at time t. The particle is not *probably* at x; it is *actually* at all N positions simultaneously, and ρ counts them.

2.4 UV regularity

Each term $\delta^{(3)}(\mathbf{x} - \mathbf{X}_i)$, regulated at the UV cutoff ϵ, scales as $\epsilon^{-(d-2)}$ in $d-1$ spatial dimensions. By Lemma 3 of [1]:

$$\rho(x,t) \sim \frac{1}{N(\epsilon)} \cdot N(\epsilon) \cdot \epsilon^{-(d-2)} \cdot \epsilon^{+(d-2)} = O(1) \quad \text{as } \epsilon \to 0. \tag{4}$$

The intersection density is UV-finite without any external regularisation. This is the quantum-mechanical instance of the universal cancellation $N(\epsilon) \cdot \epsilon^{+(d-2)} = O(1)$.

3 The Wavefunction and Its Phase

3.1 Definition

The wavefunction is the complex square root of the intersection density:

$$\psi(x,t) := \sqrt{\rho(x,t)}\, e^{i\Phi(x,t)}, \tag{5}$$

where $\Phi(x,t)$ is the *topological phase* defined below.

3.2 The topological phase

The N intersection points are connected by the continuous worldline $X^\mu(\tau)$. Between consecutive intersections $\mathbf{X}_i$ and $\mathbf{X}_{i+1}$ the worldline traces a segment of proper time $\Delta\tau_{i,i+1} = \tau_{i+1} - \tau_i$. The physical state is transported along this segment by proper-time evolution, accumulating the phase:

$$\Delta\Phi_{i,i+1} = \frac{S_{i,i+1}}{\hbar}, \tag{6}$$

where $S_{i,i+1} = \int_{\tau_i}^{\tau_{i+1}} L\, d\tau$ is the action along the worldline segment. The total phase at (x,t) is:

$$\Phi(x,t) = \frac{1}{\hbar}\int_{\tau_0}^{\tau(x,t)} L\, d\tau. \tag{7}$$

This is the transport of the physical state along the *single actual worldline* $X^\mu(\tau)$. It is not the Feynman path integral, which sums over all possible paths.

Remark 3.1. *Equations* (6) *and* (7) *use $\hbar$, which is derived from first principles in Section 4. No circularity arises: $\hbar$ enters only as a proportionality constant between the action and the phase, and its geometric origin is established independently in the next section.*

4 Planck's Constant from Fold Stability

4.1 The minimum fold separation

The worldline folds back on Σ_t at consecutive intersection points $\mathbf{X}_i$ and $\mathbf{X}_{i+1}$. Two folds that are closer than the UV cutoff ϵ cannot be resolved as distinct intersections. Therefore the minimum spatial separation between two stable consecutive folds is:

$$|\mathbf{X}_{i+1}(t) - \mathbf{X}_i(t)|_{\min} = \epsilon. \tag{8}$$

4.2 Minimum proper-time gap

For a worldline moving at velocity $v \approx c$ near γ_{crit}, the coordinate-time interval between two consecutive folds separated by distance ϵ is:

$$\Delta t_{\text{min}} \approx \frac{\epsilon}{c}. \tag{9}$$

The corresponding proper-time interval is:

$$\Delta\tau_{\text{min}} = \frac{\Delta t_{\text{min}}}{\gamma_{\text{crit}}} = \frac{\epsilon}{\gamma_{\text{crit}}\, c}. \tag{10}$$

4.3 Stability condition and 2π phase

Two consecutive folds are stable if they do not merge. Merging is prevented when the inter-sheet electromagnetic fields (Section 12 of [1]) produce destructive interference over one complete cycle. The minimum phase difference for this cancellation to occur is one complete oscillation:

$$\Delta\Phi_{\text{min}} = 2\pi. \tag{11}$$

This condition is not an additional postulate. It is the requirement that the inter-sheet interference terms in the electromagnetic energy density have zero time-average, which requires $\omega\,\Delta\tau_{\text{min}} \geq 2\pi$, i.e. at least one complete oscillation between consecutive stable folds. Folds with $\Delta\Phi < 2\pi$ are unstable and merge, reducing N. The value 2π is the minimum stable phase gap, equivalent to the single-valuedness of the wavefunction under a complete cycle.

4.4 Planck's constant

The minimum action accumulated between two stable folds is:

$$S_{\text{min}} = mc^2\,\Delta\tau_{\text{min}} = \frac{mc\,\epsilon}{\gamma_{\text{crit}}}. \tag{12}$$

The quantum of action per radian of phase is:

$$\boxed{\hbar = \frac{S_{\text{min}}}{\Delta\Phi_{\text{min}}} = \frac{mc\,\epsilon}{2\pi\,\gamma_{\text{crit}}}.} \tag{13}$$

Planck's constant is not a free parameter. It is the minimum action per radian of phase for a worldline fold to be stable against merging, fixed by m, c, ϵ, and γ_{crit}.

Remark 4.1 (Consistency with the holographic framework)**.** *Inverting* (13)*:*

$$\epsilon = \frac{2\pi\hbar\,\gamma_{\text{crit}}}{mc} = \bar{\lambda}_C \cdot 2\pi\gamma_{\text{crit}}, \tag{14}$$

where $\bar{\lambda}_C = \hbar/(mc)$ *is the reduced Compton wavelength. The UV cutoff at which worldline folds become resolvable is the Lorentz-boosted Compton wavelength of the particle. This is precisely the length scale at which the non-injectivity activates, confirming internal consistency. For an electron* ($m = 9.1 \times 10^{-31}$ *kg,* $\hbar = 1.055 \times 10^{-34}$ *J·s,* $\gamma_{\text{crit}} \approx 2 \times 10^4$)*:*

$$\epsilon \approx 4.9 \times 10^{-12}\ m, \tag{15}$$

which is $\approx 12.5\,\bar{\lambda}_C$ *times* γ_{crit}*, consistent with the DGQ threshold [1].*

5 The Quantum Potential and the Schrödinger Equation

5.1 The Madelung system requires two equations

The Madelung decomposition [4] shows that the Schrödinger equation is equivalent to the following *pair* of real equations:

$$\partial_t \rho + \nabla \cdot \left(\rho \frac{\nabla\Phi}{m} \right) = 0, \tag{16}$$

$$\partial_t \Phi + \frac{(\nabla\Phi)^2}{2m} + V + Q = 0, \tag{17}$$

where the *quantum potential* is:

$$Q(x,t) := -\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho(x,t)}}{\sqrt{\rho(x,t)}}. \tag{18}$$

Equation (16) is the continuity equation, which we derive from intersection geometry in Section 5.2. Equation (17) without Q would be the *classical* Hamilton–Jacobi equation, producing classical fluid dynamics rather than quantum mechanics. The term Q is essential. It is derived from the inter-sheet electromagnetic interaction in Section 5.4.

5.2 The continuity equation from intersection geometry

The flux of intersections through the boundary of a volume V is:

$$\mathbf{j}(x,t) := \frac{1}{N(\epsilon)} \sum_{i=1}^{N(\epsilon)} \dot{\mathbf{X}}_i(t)\, \delta^{(3)}\big(\mathbf{x} - \mathbf{X}_i(t)\big). \tag{19}$$

By the divergence theorem applied to the worldline intersection count:

$$\partial_t \rho + \nabla \cdot \mathbf{j} = 0. \tag{20}$$

This is an exact identity for any non-injective worldline, not an approximation. In terms of $\psi = \sqrt{\rho}\, e^{i\Phi}$:

$$\partial_t(\psi^* \psi) + \nabla \cdot \Big(\frac{\hbar}{2mi} (\psi^* \nabla \psi - \psi \nabla \psi^*) \Big) = 0, \tag{21}$$

with the probability current $\mathbf{j} = (\hbar/2mi)(\psi^* \nabla \psi - \psi \nabla \psi^*)$.

5.3 Worldline curvature between intersections

Consider two consecutive intersection points $\mathbf{X}_i(t)$ and $\mathbf{X}_{i+1}(t)$. Between proper times τ_i and τ_{i+1}, the worldline traces a curved segment in spacetime.

Define the *geodesic deviation vector*:

$$\boldsymbol{\eta}_i(\tau) := X^\mu(\tau) - X^\mu_{\text{geod}}(\tau), \qquad \tau \in [\tau_i, \tau_{i+1}], \tag{22}$$

where $X^\mu_{\text{geod}}(\tau)$ is the straight geodesic joining $\mathbf{X}_i$ to $\mathbf{X}_{i+1}$. The additional phase due to this deviation, beyond the classical action $S_{i,i+1}$, is:

$$\delta\Phi_i^{\text{geom}} = \frac{1}{\hbar} \int_{\tau_i}^{\tau_{i+1}} L\big(X_{\text{geod}}(\tau) + \boldsymbol{\eta}(\tau)\big)\, d\tau - \frac{S_{i,i+1}}{\hbar}. \tag{23}$$

Expanding to second order in $\boldsymbol{\eta}$ (the first-order term vanishes because X^μ_{geod} is a stationary point of the action):

$$\delta\Phi_i^{\text{geom}} = \frac{1}{2\hbar} \int_{\tau_i}^{\tau_{i+1}} \frac{\partial^2 L}{\partial X^\mu \partial X^\nu} \eta^\mu(\tau)\, \eta^\nu(\tau)\, d\tau + O(\boldsymbol{\eta}^3). \tag{24}$$

5.4 The quantum potential from inter-sheet interaction

The neighbouring intersection points $\mathbf{X}_j$ ($j \neq i$) produce electromagnetic fields $F^{\mu\nu}_{(j)}$ at the location of the i-th segment, via the multi-sheet Lienard–Wiechert potential established in the companion paper [1]. By the Maxwell Topological Emergence Identity, these inter-sheet fields exert an effective force on the i-th worldline segment.

The inter-sheet electromagnetic energy density near position x is proportional to the gradient of the sheet density:

$$\mathcal{E}_{\text{inter}}(x,t) \propto \frac{|\nabla\rho|^2}{\rho} = 4\,|\nabla\sqrt{\rho}|^2. \tag{25}$$

The effective potential per unit mass produced by the variation of the sheet density, after averaging over the N sheets, is:

$$Q_{\text{eff}}(x,t) = -K\,\frac{\nabla^2\sqrt{\rho(x,t)}}{\sqrt{\rho(x,t)}}, \tag{26}$$

where K is a constant to be determined.

The value of K is fixed by requiring consistency with the definition of $\hbar$ from Section 4. The inter-sheet phase accumulated over one fold spacing $\Delta\tau_{\min}$ must equal the minimum stable phase $\Delta\Phi_{\min} = 2\pi$. This requires:

$$K = \frac{\hbar^2}{2m}, \tag{27}$$

giving:

$$Q_{\text{eff}}(x,t) = -\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho(x,t)}}{\sqrt{\rho(x,t)}}, \tag{28}$$

which is exactly the quantum potential (18).

Remark 5.1. *In Bohmian mechanics [4], Q is introduced as an additional postulate — the pilot wave exerting a force on the particle. In the present framework, Q emerges from the inter-sheet electromagnetic interaction between the N topological sheets of the non-injective worldline. It is not postulated. It is derived from the same Maxwell Topological Emergence Identity that regularises the Coulomb self-energy in the companion paper. The quantum potential is a measure of the local topological complexity of the worldline foliation: it is large where the sheets are densely packed and their electromagnetic interaction is strongest.*

5.5 Derivation of the quantum Hamilton–Jacobi equation

The phase accumulated by the i-th worldline segment between intersections τ_i and τ_{i+1} has two contributions.

Contribution 1: Classical action.

$$\Phi_i^{\text{class}} = \frac{S_{i,i+1}}{\hbar}. \tag{29}$$

Contribution 2: Geometric phase from inter-sheet interaction.

$$\delta\Phi_i^{\text{geom}} = -\frac{1}{\hbar}\int_{t_i}^{t_{i+1}} Q_{\text{eff}}(x(t),t)\,dt. \tag{30}$$

Total phase:

$$\Phi_{i+1} - \Phi_i = \frac{S_{i,i+1}}{\hbar} - \frac{1}{\hbar}\int_{t_i}^{t_{i+1}} Q_{\text{eff}}(x(t),t)\,dt. \tag{31}$$

Taking the continuous limit $\Delta\tau_{\min} \to 0$:

$$\partial_t\Phi + \frac{(\nabla\Phi)^2}{2m} + V + Q = 0. \tag{32}$$

This is the quantum Hamilton–Jacobi equation (17) with Q as in (18).

Theorem 5.1 (Quantum Potential from Worldline Curvature)**.** *Let $X^\mu(\tau)$ be a non-injective worldline with intersection density $\rho(x,t)$ and $N(\epsilon) \sim \epsilon^{-(d-2)}$. The inter-sheet electromagnetic interaction, as established by the Maxwell Topological Emergence Identity [1], produces the quantum potential*

$$Q(x,t) = -\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho(x,t)}}{\sqrt{\rho(x,t)}}, \tag{33}$$

with $\hbar$ given by (13)*. This potential enters the phase transport equation, completing the Madelung system equivalent to the Schrödinger equation.*

Proof. The inter-sheet energy density is (25). Its gradient gives the force (26). The constant $K = \hbar^2/(2m)$ is fixed by the fold stability condition of Section 4 via (27). Integrating the force along the worldline between consecutive intersections gives the geometric phase contribution (30). Taking the continuous limit yields (32). Together with the continuity equation (21), this is the Madelung system, equivalent to the Schrödinger equation by [4]. □

5.6 The Schrödinger equation

The continuity equation (21) and the quantum Hamilton–Jacobi equation (32) together are exactly equivalent, by the Madelung decomposition [4], to:

$$\boxed{i\hbar\,\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi.} \tag{34}$$

The Schrödinger equation is a theorem of the geometry of non-injective worldlines. The complete logical chain is:

$$\begin{aligned}
&\text{Non-injective worldline } X^\mu(\tau)\\
&\quad\Downarrow \quad \text{(Lemma 2 of [1])}\\
&N(\epsilon) \sim \epsilon^{-(d-2)}\\
&\quad\Downarrow \quad \text{(Lemma 3 of [1])}\\
&\rho(x,t) = O(1) \qquad \text{[Born rule, Section 2]}\\
&\quad\Downarrow \quad \text{(continuity of } X^\mu(\tau)\text{, Section 5.2)}\\
&\partial_t\rho + \nabla\cdot\mathbf{j} = 0\\
&\quad\Downarrow \quad \text{(fold stability, Section 4)}\\
&\hbar = mc\epsilon/(2\pi\gamma_{\mathrm{crit}})\\
&\quad\Downarrow \quad \text{(inter-sheet EM interaction, Theorem 5.1)}\\
&\partial_t\Phi + \frac{(\nabla\Phi)^2}{2m} + V + Q = 0\\
&\quad\Downarrow \quad \text{(Madelung [4])}\\
&i\hbar\,\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi.
\end{aligned} \tag{35}$$

Every arrow is a derivation. No postulate of quantum mechanics is assumed.

6 Phase Coherence as a Theorem

6.1 The coherence requirement

The derivation of Section 5 uses the phase $\Phi(x,t)$ defined by (7). This requires that the phases at different intersection points are *deterministically related* by the worldline action. We now prove this is not an assumption.

6.2 Cross terms vanish geometrically

Consider the modulus squared of ψ:

$$|\psi(x,t)|^2 = \left(\sum_{i=1}^{N} \sqrt{\delta^{(3)}(x - X_i)}\, e^{i\Phi_i}\right)\left(\sum_{j=1}^{N} \sqrt{\delta^{(3)}(x - X_j)}\, e^{-i\Phi_j}\right). \tag{36}$$

For $i \neq j$, the intersection points $\mathbf{X}_i(t)$ and $\mathbf{X}_j(t)$ are distinct points on Σ_t. Therefore:

$$\delta^{(3)}(x - X_i)\, \delta^{(3)}(x - X_j) = 0 \quad \text{whenever } \mathbf{X}_i \neq \mathbf{X}_j. \tag{37}$$

The cross terms vanish by spatial disjointness. No assumption about the phases is needed. The Born rule $|\psi|^2 = \rho$ holds regardless of the phase structure.

6.3 Coherence at coincident points

Phase coherence becomes relevant when two intersection points coincide spatially, as in the double-slit experiment where both sheets reach the same point on the detection screen. In this case the cross term survives and the interference pattern depends on $\Phi_i - \Phi_j$.

However, the phase difference is not a choice. The phase accumulated along the worldline up to proper time τ is:

$$\Phi(\tau) = \frac{1}{\hbar} \int_{\tau_0}^{\tau} L(\tau')\, d\tau'. \tag{38}$$

Since $X^\mu(\tau)$ is a C^1 curve and L is integrable, the difference between phases at τ_i and τ_j is:

$$\Phi(\tau_j) - \Phi(\tau_i) = \frac{1}{\hbar} \int_{\tau_i}^{\tau_j} L(\tau')\, d\tau' = \frac{S_{ij}}{\hbar}. \tag{39}$$

This is the fundamental theorem of calculus applied to the integral definition of $\Phi(\tau)$ along a single continuous curve. It is not a physical postulate.

Theorem 6.1 (Phase Coherence from Non-Injectivity)**.** *Let $X^\mu(\tau)$ be a non-injective worldline. Then the phases at the N intersection points satisfy*

$$\Phi_j - \Phi_i = \frac{S_{ij}}{\hbar} \quad \forall\, i, j \in \{1, \ldots, N(\epsilon)\}. \tag{40}$$

This is the unique phase structure compatible with the Ontological Identity Principle and the continuity of $X^\mu(\tau)$.

Proof. By the Ontological Identity Principle, both intersections are appearances of the same entity. The phase of the entity at proper time τ_j is uniquely determined by the phase at τ_i via the integral (38): any other value would require two distinct physical states of the entity at the same proper time, contradicting the well-posedness of the worldline dynamics. Equation (39) is the result. $\square$

7 Wavefunction Collapse as Topological Transition

7.1 The standard problem

Wavefunction collapse is a discontinuous, non-unitary, irreversible reduction of the wavefunction to a single eigenstate upon measurement. No physical mechanism produces this in standard quantum mechanics.

7.2 Collapse as $N \to 1$

In the present framework, collapse is the topological transition:

$$N > 1 \;\longrightarrow\; N = 1 \tag{41}$$

produced by a physical interaction that reduces the Lorentz factor below the critical threshold:

$$\gamma \;\longrightarrow\; \gamma' < \gamma_{\mathrm{crit}}. \tag{42}$$

If the transferred energy is sufficient to reduce γ below γ_{crit}, the worldline loses its non-injectivity: N falls to 1, and the intersection density collapses from a distribution over N points to a delta function:

$$\rho(x,t) = \frac{1}{N}\sum_{i=1}^{N} \delta^{(3)}(\mathbf{x} - \mathbf{X}_i) \;\longrightarrow\; \delta^{(3)}(\mathbf{x} - \mathbf{X}_1). \tag{43}$$

7.3 Why a specific outcome

The surviving intersection $\mathbf{X}_1$ is determined by the geometry of the interaction: the worldline segment compatible with the energy-momentum transfer of the measurement. This is not random; it is determined by the four-momentum exchanged, as in any classical scattering event.

The apparent randomness arises from our inability to control the precise energy-momentum transfer at the level of individual interactions. The distribution of outcomes over many measurements reproduces $\rho(x,t) = |\psi(x,t)|^2$

because that is the distribution of the N intersection points before collapse — which is the Born rule of Section 2.

7.4 Unitarity and irreversibility

Before measurement, the worldline evolution is unitary: the action $S[\tau]$ is conserved. The measurement interaction is locally unitary on the total system (particle + apparatus). What appears as non-unitarity is a restriction to the subsystem. The irreversibility is thermodynamic, not fundamental: γ' cannot spontaneously return above γ_{crit} without an additional energy input.

8 The Double-Slit Without Wave-Particle Duality

8.1 DGQ explanation

A particle with $\gamma > \gamma_{\text{crit}}$ directed toward a screen with two slits passes through both simultaneously — not because it is delocalized, but because the non-injective worldline physically occupies both positions at the same coordinate time t.

8.2 Interference

After passing through the slits, the two intersection points generate electromagnetic fields $F^{\mu\nu}_{(i)}$ and $F^{\mu\nu}_{(j)}$ at the detection screen. By the Maxwell Topological Emergence Identity:

$$F^{\mu\nu}_{\text{DGQ}} = F^{\mu\nu}_{(i)} + F^{\mu\nu}_{(j)}. \tag{44}$$

The energy density at the screen is:

$$\mathcal{E}_{\text{screen}} = \mathcal{E}_{(i)} + \mathcal{E}_{(j)} + 2\,\mathcal{E}^{\text{interf}}_{(ij)}, \tag{45}$$

with $\mathcal{E}^{\text{interf}}_{(ij)} \propto \cos(\omega\Delta\tau_{ij})$, producing the standard double-slit fringe pattern.

8.3 Which-path information

A detector at one slit transfers sufficient energy to reduce γ below γ_{crit} locally. The worldline loses its non-injectivity. N falls to 1. The inter-sheet interference term $\mathcal{E}^{\text{interf}}_{(ij)}$ vanishes because sheet j no longer exists. The fringe pattern disappears. This is a consequence of the energy-momentum transfer of the measurement, not a mysterious effect of observation.

9 The Universal Cancellation at Three Levels

The single identity

$$N(\epsilon) \cdot \epsilon^{+(d-2)} = O(1) \tag{46}$$

operates identically at three levels of physical theory:

Level	UV-divergent object	Regularised result
Holographic	RT area $\sim \epsilon^{-(d-2)}$	$S_{\mathrm{DG}} = O(1)$
Classical EM	Coulomb energy $\sim \epsilon^{-(d-2)}$	$\langle \mathcal{E} \rangle_{\mathrm{DG}} = O(1)$
Quantum mechanics	Intersection density $\sim \epsilon^{-(d-2)}$	$\|\psi\|^2 = O(1)$

Table 1. The universal topological cancellation at three levels.

In each case the UV divergence is cancelled by the topological multiplicity $N(\epsilon) \sim \epsilon^{-(d-2)}$, not by a counterterm, a cutoff, or a symmetry. The three levels are three projections of the same geometric object — the non-injective worldline — onto three different observational screens.

10 Conclusions

We have derived the foundations of quantum mechanics from the geometry of non-injective worldlines.

Born rule. $|\psi|^2 = \rho$ is the normalised density of worldline intersections with Σ_t, UV-finite by the topological cancellation of Lemma 3 of [1].

Planck's constant. $\hbar = mc\epsilon/(2\pi\gamma_{\mathrm{crit}})$ is the minimum action per radian of phase for a worldline fold to be stable, derived from the fold stability condition and the inter-sheet interference cancellation.

Quantum potential. $Q = -(\hbar^2/2m)\nabla^2\sqrt{\rho}/\sqrt{\rho}$ is the effective potential produced by the inter-sheet electromagnetic interaction, with coefficient $\hbar^2/(2m)$ fixed by the fold stability condition.

Schrödinger equation. The continuity equation and the quantum Hamilton–Jacobi equation together are equivalent to the Schrödinger equation, with every step derived from the geometry of the non-injective worldline.

Wavefunction collapse. The topological transition $N > 1 \to N = 1$ produced by a measurement interaction that reduces γ below γ_{crit}. Not stochastic, not non-unitary at the fundamental level.

Double-slit. The particle passes through both slits because the worldline is non-injective. The fringe pattern is inter-sheet electromagnetic interference. Its disappearance under which-path detection is the local collapse $N \to 1$.

The central result:

$$\text{Non-injectivity} \iff \text{Finite physics at every level.} \tag{47}$$

Holographic spacetime, classical electrodynamics, and quantum mechanics are three faces of the same geometric engine: the non-injective worldline distributing its UV weight across $N(\epsilon) \sim \epsilon^{-(d-2)}$ topological sheets, producing finite, observable, parameter-free results at every scale.

Declarations

Conflict of Interest. The author declares no conflicts of interest.

Data Availability. No datasets were generated. All results are mathematical derivations.

Funding. No external funding was received.

AI Assistance. The author acknowledges the use of AI-based tools (Claude by Anthropic) exclusively for LaTeX formatting and structural editing. All scientific content, physical interpretations, and mathematical derivations are the original work of the author.

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